Summation formulae for the product of the *q*-Kummer functions from $E_q(2)$

2001 J. Phys. A: Math. Gen. 34 1333

(http://iopscience.iop.org/0305-4470/34/7/308)

INSTITUTE OF PHYSICS PUBLISHING JOURNAL OF PHYSICS A: MATHEMATICAL AND GENERAL

J. Phys. A: Math. Gen. **34** (2001) 1333–1340 www.iop.org/Journals/ja PII: S0305-4470(01)16737-5

Summation formulae for the product of the q-Kummer functions from $E_q(2)$

H Ahmedov[1] **and I H Duru**[1,2]

[1] Feza Gürsey Institute, PO Box 6, 81220, Çengelköy, Istanbul, Turkey
[2] Trakya University, Mathematics Department, PO Box 126, Edirne, Turkey

E-mail: hagi@gursey.gov.tr and duru@gursey.gov.tr

Received 29 August 2000, in final form 21 November 2000

Abstract

Using the representation of $E_q(2)$ on the non-commutative space $zz^* - qz^*z = \sigma$; $q < 1$, $\sigma > 0$ summation formulae for the product of two, three and four q-Kummer functions are derived.

PACS numbers: 0230G, 0220

1. Introduction

Properties of manifolds can be investigated by means of their automorphism groups. Non-commutative spaces are studied similarly. For example, the quantum groups $E_q(2)$ and $SU_q(2)$ are the symmetry groups of the quantum plane and the quantum sphere, respectively [1–3]. The group representation theory gives the possibility of constructing the complete set of orthogonal functions on these deformed spaces. For example the Hahn–Exton q-Bessel and q-Legendre functions appears as the matrix elements of the unitary representations of $E_q(2)$ [4–7] and $SU_q(2)$ [8, 9], which are the complete set of orthogonal functions on the quantum plane and the quantum sphere, respectively. Using group-theoretical methods the invariant distance and the Green functions have also been written on the quantum sphere [10] and the quantum plane [11].

In recent works we have studied the non-commutative space $[z, z^*] = \sigma$ (i.e. the space generated by the Heisenberg algebra) by means of its automorphism groups $E(2)$ and $SU(1, 1)$ [12, 13]. The basis in this non-commutative space where irreducible representations of $E(2)$ are realized were found to be the Kummer functions which involves the coordinates z, z^* not as their arguments but as indices. That study enables us to obtain generic summation formulae involving Kummer and Bessel functions. For the $SU(1, 1)$ case the basis is given in terms of the hypergeometric functions having the non-commutative coordinates z and z^* as the parameters. Again we derived generic summation formulae involving hypergeometric and Jacobi functions. This analysis enables us to construct different complete sets of orthogonal functions on the non-commutative space. Both studies also provide new group-theoretical interpretations for the already known relations involving special functions.

Motivated by the outcomes of the above-mentioned studies, in this paper we consider the two-parametric deformation of the plane which is the $*$-algebra P_q^σ generated by z and z^* with

$$zz^* - qz^*z = \sigma \qquad q < 1 \quad \sigma > 0 \tag{1}$$

which possesses the symmetry of the group $E_q(2)$. In the $\sigma \to 0$ limit it becomes the usual quantum plane. In the $q \to 1$ limit it becomes the algebra of functions on the Heisenberg algebra. This study allows us to obtain many identities involving several Hahn–Exton q-Bessel and Moak q-Laguerre functions which are the special forms of the q-Kummer functions. Note that previously some formulae involving q-Laguerre functions were derived by making use of the representation theory of the q-oscillator algebra [14–19]. Some relations involving the basic Bessel and Laguerre functions were also considered in [21].

In section 2 we realize $E_q(2)$ as the automorphism group of the non-commutative space P_q^σ. In section 3 we construct the basis in P_q^σ where the irreducible representations of $E_q(2)$ are realized. Section 4 is devoted to the generic summation formulae for the product of two, three and four q-Kummer functions. In section 5 some simple examples are presented.

2. $E_q(2)$ as the symmetry group of P_q^σ

The quantum group $E_q(2)$ is the $*$-Hopf algebra generated by B, B^* and A with relations

$$BB^* = qB^*B \qquad AB = qBA \qquad AB^* = qB^*A \qquad A^* = A^{-1} \tag{2}$$

coalgebra operations

$$\Delta(B) = B \otimes 1 + A \otimes B \qquad \Delta(A) = A \otimes A \tag{3}$$

and antipode map

$$S(B) = -A^{-1}B \qquad S(B^*) = -AB^* \qquad S(A) = A^{-1}. \tag{4}$$

The map $\delta : P_q^\sigma \longrightarrow E_q(2) \otimes P_q^\sigma$ given by

$$\delta(z) = B \otimes 1 + A \otimes z \tag{5}$$

$$\delta(z^*) = B^* \otimes 1 + A^* \otimes z^* \tag{6}$$

due to

$$\delta(z)\delta(z^*) - q\delta(z^*)\delta(z) = 1 \otimes 1 \tag{7}$$

and

$$(\delta(z))^* = \delta(z^*) \tag{8}$$

defines the corepresentation of the Hopf algebra $E_q(2)$ in the $*$-algebra P_q^σ. Formulae

$$\begin{aligned} z|n, j\rangle &= \sqrt{(n)_q}|n-1, j\rangle \\ z^*|n, j\rangle &= \sqrt{(n+1)_q}|n+1, j\rangle \\ B|n, j\rangle &= q^{j/2}|n, j-1\rangle \\ B^*|n,_j\rangle &= q^{(j+1)/2}|n, j+1\rangle \\ A|n, j\rangle &= |n, j-2\rangle \end{aligned} \tag{9}$$

where

$$(n)_q = \frac{1-q^n}{1-q}$$

define the $*$-representation of the algebra $E_q(2) \otimes P_q^\sigma$ in some suitable domain of the Hilbert space X with the basis $\{|n, j\rangle\}$, $n = 0, 1, 2, \ldots$ and $j \in \mathbb{Z}$. In the above formula we have put $\sigma = 1$. When we need to calculate $\sigma \to 0$ limit we replace z, z^* by $z/\sqrt{\sigma}, z^*/\sqrt{\sigma}$.

Let us define in X a new basis such that

$$\delta(z)|n, j\rangle' = \sqrt{(n)_q}|n-1, j\rangle' \tag{10}$$

$$\delta(z^*)|n, j\rangle' = \sqrt{(n+1)_q}|n+1, j\rangle'. \tag{11}$$

Due to

$$z\mathrm{e}_q^{-xz^*} = -x\mathrm{e}_q^{xz^*} + \mathrm{e}_q^{-qxz^*}z \tag{12}$$

with

$$\mathrm{e}_q^x = \sum_{k=0}^{\infty} \frac{x^k}{(k)_q!} \tag{13}$$

being the q-deformed exponential function, we conclude that

$$|0, j\rangle' = \mathrm{e}_q^{-A^*Ba^*}\sqrt{\mathrm{e}_q^{-B^*B}}|0, j\rangle \tag{14}$$

is the ground state of the new basis

$$\delta(z)|0, j\rangle' = 0. \tag{15}$$

Applying the creation operator $(\delta(z))^*$ on this state we can generate the desired basis in X:

$$|n, j\rangle' = \frac{(\delta(z^*))^n}{\sqrt{(n)_q!}}|0, j\rangle'. \tag{16}$$

We also have

$$\delta(z) = UzU^* \tag{17}$$

where U is the unitary operator in

$$|n, j\rangle' = U|n, j\rangle. \tag{18}$$

Before closing this section we give the explicit formula for the matrix representation of U:

$$U_{(mi)(nj)} = \langle m, i|n, j\rangle'. \tag{19}$$

For $|n, j\rangle = |n\rangle|j\rangle$ we first define

$$U_{mn} = \langle n|\delta(z^*))^n \mathrm{e}_q^{-A^*Bz^*}|0\rangle\sqrt{\frac{\mathrm{e}_q^{-B^*B}}{(n)_q!}} \tag{20}$$

which is the function of B, B^*, A and A^*. Then

$$U_{(mi)(nj)} = \langle i|U_{mn}|j\rangle. \tag{21}$$

After some algebra we obtain

$$U_{mn} = A^{-m}B^{*n-m}\Phi_{mn}(\eta) \qquad \text{for} \quad n \geqslant m \tag{22}$$

and

$$U_{mn} = q^{(m-n)(m-n-1)/2}A^{-m}(-B)^{m-n}\Phi_{nm}(\eta) \qquad \text{for} \quad m \geqslant n \tag{23}$$

where $\eta^2 \equiv B^*B$ and

$$\Phi_{mn}(\eta) = \sqrt{\frac{(n)_q!}{(m)_q!}}\frac{\sqrt{e_q^{-\eta^2}}}{(n-m)_q!}\Phi^q(q^{-m}, q^{1+n-m}; q^{n+1}\eta^2). \tag{24}$$

Here

$$\Phi^q(a, b; x) = \sum_{k=0}^{\infty}\frac{q^{k(k-1)/2}(a; q)_k}{(q; q)_k(b; q)_k}((1-q)x)^k \tag{25}$$

which in the $q \to 1$ limit reduces to the Kummer function:

$$\lim_{q\to 1}\Phi^q(q^c, q^d; x) = \Phi(c, d; x). \tag{26}$$

We call it the q-Kummer function. The functions Φ_{nm} can also be expressed in terms of Moak's q-Laguerre polynomials [22]

$$L_n^{q(\alpha)}(x) = \frac{(q^{1+\alpha}; q)_n}{(q; q)_n}\Phi^q(q^{-n}, q^{1+\alpha}; q^{1+\alpha+n}x) \tag{27}$$

as

$$\Phi_{mn}(\eta) = \sqrt{e_q^{-\eta^2}\frac{(m)_q!}{(n)_q!}}L_m^{q(n-m)}(\eta^2) \qquad \text{for} \quad n \geqslant m. \tag{28}$$

3. Irreducible representations of $E_q(2)$ in P_q^σ

The deformed enveloping algebra $U_q(e(2))$ is the $*$-Hopf algebra generated by P, P^* and K with relations

$$P^*P = qPP^* \qquad KP = qPK \qquad P^*K = qKP^* \qquad K^* = K \tag{29}$$

$$\Delta(P) = P\otimes 1 + K\otimes P \qquad \Delta(K) = K\otimes K \tag{30}$$

$$S(P) = -K^{-1}P \qquad S(P^*) = -K^{-1}P^* \qquad S(K) = K^{-1}. \tag{31}$$

The duality pairing between $E_q(2)$ and $U_q(e(2))$ is

$$\begin{aligned}
\langle P, B^{*n}B^mA^j\rangle &= \mathrm{i}\delta_{m1}\delta_{n0} \\
\langle P^*, B^{*n}B^mA^j\rangle &= \mathrm{i}\delta_{m0}\delta_{n1} \\
\langle K, B^{*n}B^mA^j\rangle &= q^j\delta_{m0}\delta_{n0}.
\end{aligned} \tag{32}$$

The formula

$$R(X)F = \sum_j \langle X, F_j\rangle F'_j \qquad F \in P_q^\sigma \tag{33}$$

where

$$\delta(F) = \sum_j F_j \otimes F'_j \tag{34}$$

defines the right representation of $U_q(e(2))$ in P_q^σ. We have

$$\begin{aligned}
R(K)z^{*n}z^m &= q^{m-n}z^{*n}z^m \\
R(P)z^{*n}z^m &= \mathrm{i}q^{-n}(m)_q z^{*n}z^{m-1} \\
R(P^*)z^{*n}z^m &= \mathrm{i}q^{-n+1}(n)_q z^{*n-1}z^m.
\end{aligned} \tag{35}$$

The unitary irreducible representation of $U_q(e(2))$ is defined by the weight $\lambda \in \mathbb{R}$ and is given by [4, 6, 7]

$$\pi(K)e_j = q^j e_j \qquad \pi(P)e_j = \lambda q^{j/2} e_{j-1} \qquad \pi(P^*)e_j = \lambda q^{(j+1)/2} e_{j+1} \tag{36}$$

where e_j is some orthogonal basis. We look for the basis $D_j^\lambda(z, z^*)$ in P_q^σ on which the irreducible representation of $U_q(e(2))$ is realized; that is,

$$R(K)D_j^\lambda(z, z^*) = q^j D_j^\lambda(z, z^*) \tag{37}$$

$$R(P)D_j^\lambda(z, z^*) = \lambda q^{j/2} D_{j-1}^\lambda(z, z^*) \tag{38}$$

$$R(P^*)D_j^\lambda(z, z^*) = \lambda q^{(j+1)/2} D_{j+1}^\lambda(z, z^*). \tag{39}$$

Equation (37) implies

$$D_j^\lambda(z, z^*) = \begin{cases} f_j^\lambda(\zeta) z^j & \text{for} \quad j \geqslant 0 \\ z^{*-j} f_{-j}^\lambda(\zeta) & \text{for} \quad j \leqslant 0 \end{cases} \tag{40}$$

where

$$\zeta \equiv 1 - (1-q)z^* z. \tag{41}$$

Inserting the ansatz (40) into (38) and (39) we obtain

$$f_j^\lambda(\zeta) = \frac{q^{j^2/4}(\mathrm{i}\lambda)^j}{(j)_q!} \Phi^q(\zeta^{-1}, q^{j+1};\ q^{j+1}\lambda^2\zeta). \tag{42}$$

In the derivation of (42) we used

$$z^{*n} z^n = (-)^n q^{n(1-n)/2} (\zeta^{-1}; q)_n \zeta^n. \tag{43}$$

By means of the universal T-matrix [6, 20] we can exponentiate (37) and obtain

$$\delta(D_j^\lambda) = \sum_{i=-\infty}^{\infty} t_{ji}^\lambda D_i^\lambda \tag{44}$$

where

$$t_{ij}^\lambda = q^{(i^2-j^2)/4} \frac{(\mathrm{i}\lambda B)^{i-j} A^j}{(i-j)_q!} \Phi^q(0, q^{1+i-j}; (q-1)q^{1-j}(\lambda\eta)^2) \qquad \text{for} \quad i \geqslant j \tag{45}$$

$$t_{ij}^\lambda = q^{(i^2-j^2)/4} \frac{A^j (\mathrm{i}\lambda B^*)^{j-i}}{(j-i)_q!} \Phi^q(0, q^{1+j-i}; (q-1)q^{1-i}(\lambda\eta)^2) \qquad \text{for} \quad j \geqslant i \tag{46}$$

are the matrix elements of the irreducible representations of $E_q(2)$ [6, 7]. We can express them in terms of the Hahn–Exton q-Bessel functions [23]

$$J_k^q(x) = \frac{x^k}{(k)_q!} \Phi^q(0; q^{1+k}|q; (q-1)qx^2) \tag{47}$$

as

$$t_{ij}^\lambda = (\sqrt{-1} q^{\frac{1}{4}})^{i-j} V^{i-j} A^j J_{i-j}^q(q^{-j/2}\lambda\eta) \qquad \text{for} \quad i \geqslant j \tag{48}$$

$$t_{ij}^\lambda = (\sqrt{-1} q^{-\frac{1}{4}})^{j-i} V^{i-j} A^j J_{j-i}^q(q^{-i/2}\lambda\eta) \qquad \text{for} \quad j \geqslant i \tag{49}$$

where V is the unitary operator defined by $B = V\eta$.

In the $\sigma \to 0$ limit the non-commutative space P_q^σ becomes the quantum plane $E_q(2)/U(1)$ generated by B, B^*:

$$\lim_{\sigma\to 0} D_j^{\sqrt{\sigma}\lambda}\left(\frac{B}{\sqrt{\sigma}}, \frac{B^*}{\sqrt{\sigma}}\right) = t_{j0}^\lambda. \tag{50}$$

In the $q \to 1$ limit P_q^σ becomes the non-commutative space generated by the Heisenberg algebra [12]:

$$\lim_{q\to 1} D_j^\lambda(z, z^*) = \Phi(-zz^*;\ 1+j; \lambda^2)\frac{(\mathrm{i}\lambda z)^j}{j!}. \tag{51}$$

In the $\sigma \to 0$ and $q \to 1$ limit we arrive at the complex plane $E(2)/U(1)$:

$$\lim_{\sigma\to 0}\lim_{q\to 1} D_j^{\sqrt{\sigma}\lambda}\left(\frac{r\mathrm{e}^{\mathrm{i}\psi}}{\sqrt{\sigma}}, \frac{r\mathrm{e}^{-\mathrm{i}\psi}}{\sqrt{\sigma}}\right) = \mathrm{i}^j\mathrm{e}^{-\mathrm{i}j\psi}J_j(\lambda r). \tag{52}$$

4. Summation formulae for the q-Kummer functions

Equations (17) and (44) imply

$$UD_j^\lambda U^* = \sum_{i=-\infty}^{\infty} t_{ji}^\lambda D_i^\lambda \tag{53}$$

or

$$UD_j^\lambda = \sum_{i=-\infty}^{\infty} t_{ji}^\lambda D_i^\lambda U \tag{54}$$

$$D_j^\lambda = \sum_{i=-\infty}^{\infty} t_{ji}^\lambda U^* D_i^\lambda U. \tag{55}$$

The above formulae define the summation of products of two, three and four q-Kummer functions. Sandwiching (53)–(55) between the states $\langle m|$ and $|n\rangle$ and using

$$(D_j^\lambda)_{mn} = \sqrt{\frac{(n)_q!}{(m)_q!}} f_j^\lambda(q^m)\delta_{j,n-m} \qquad \text{for} \quad j \geqslant 0 \tag{56}$$

$$(D_{-j}^\lambda)_{mn} = (D_j^\lambda)_{nm} \tag{57}$$

we obtain

$$\sum_{s=0}^{\infty}(D_j^\lambda)_{ss+j}U_{ms}U^*_{s+jn} = (D_{n-m}^\lambda)_{mn}t_{jn-m}^\lambda \qquad \text{for} \quad j \geqslant 0 \tag{58}$$

$$\sum_{s=0}^{\infty}(D_j^\lambda)_{s-js}U_{ms-j}U^*_{sn} = (D_{n-m}^\lambda)_{mn}t_{jn-m}^\lambda \qquad \text{for} \quad j < 0 \tag{59}$$

$$\sum_{s=0}^{\infty}t_{js-m}^\lambda(D_{s-m}^\lambda)_{ms}U_{sn} = (D_j^\lambda)_{n-jn}U_{mn-j} \qquad \text{for} \quad n \geqslant j \tag{60}$$

$$\sum_{s=0}^{\infty}t_{js-m}^\lambda(D_{s-m}^\lambda)_{ms}U_{sn} = 0 \qquad \text{for} \quad n < j \tag{61}$$

and

$$\sum_{s,l=0}^{\infty} t^{\lambda}_{jl-s}(D^{\lambda}_{l-s})_{sl}U^*_{ms}U_{ln} = (D^{\lambda}_{j})_{mn}\delta_{jn-m}. \tag{62}$$

In the above formulae Us and ts are given in terms of the q-Kummer functions of the operator $\eta = B^*B$ (see (22), (23) and (45)).

In the coming section we give some simple examples.

5. Examples

A. For $n = m = 0,\ j \geqslant 0$, equation (58) implies

$$\sum_{s=0}^{\infty} \frac{q^{s(1-s)/2}\eta^{2s}}{(s)_q!}\Phi^q(q^{-s}, q^{1+j};\ q^{1+j+s}\lambda^2) = \frac{(\mathrm{i}\lambda\eta)^{-j}(j)_q!}{\sqrt{\mathrm{e}_q^{-\eta^2}\mathrm{e}_q^{-q^j\eta^2}}}J^q_j(\lambda\eta) \tag{63}$$

which in the $q \to 1$ limit gives [24] (p 1038, equation (3) of 8.975)

$$\sum_{s=0}^{\infty} \frac{\eta^{2s}}{s!}\Phi(-s, 1+j; \lambda^2) = j!(\eta\lambda)^{-j}\mathrm{e}^{\eta^2}J_j(\lambda\eta). \tag{64}$$

B. For $n = 0$ and $k \equiv -j \geqslant m$, equation (60) implies

$$\sum_{s=0}^{\infty} q^{s(s+m-2k)/2}C_{sm}\eta^s J^q_{s+k-m}(q^{(s+k)/2}\lambda\eta) = \frac{q^{k(k-m)/2}\lambda^k\eta^{k-m}}{(m)_q!(k-m)_q!}\Phi^q(q^{-m}, q^{1+k-m}; q^{1+k}\eta^2) \tag{65}$$

where

$$C_{sm} = \frac{(-)^s\lambda^{m-s}}{(s)_q!(m-s)_q!}\Phi_q(q^{-s}, q^{1+m-s}; q^{1+m}\lambda^2) \qquad \text{for} \quad m \geqslant s. \tag{66}$$

For $s \geqslant m$ one has to replace m, s with s, m on the right-hand side of the above expression. When $m = 0$ we have

$$\sum_{s=0}^{\infty} q^{\frac{1}{2}s^2+sk}\frac{(\lambda\eta)^s}{(s)_q!}J^q_{s+k}(q^{(s+k)/2}\lambda\eta) = q^{k^2/2}\frac{(\lambda\eta)^k}{(k)_q!} \tag{67}$$

which is the quantum analogue of a known formula [24, p 974, equation (1) of 8.515].

C. For $j = \lambda = 0$, equation (62) implies the unitarity condition for the operator U:

$$\sum_{s=0}^{\infty}(U_{sm})^*U_{sn} = \delta_{nm} \tag{68}$$

where we have used

$$U^*_{ms} = (U_{sm})^*. \tag{69}$$

For $n = m$ with $x = \eta^2$ we have

$$\sum_{s=0}^{n-1} \frac{q^{(n-s)(n-s+1)/2}(n)_q!x^{n-s}}{(s)_q!}(L^{q(s-n)}_s(x))^2 + \sum_{s=n}^{\infty} \frac{q^{(n-s)(n-s+1)/2}(s)_q!x^{s-n}}{(n)_q!}(L^{q(n-s)}_n(x))^2 = e^x_{q^{-1}}. \tag{70}$$

In deriving the above examples one frequently uses the identities

$$B^kB^{*k} = q^{k(k+1)/2}\eta^{2k} \qquad B^{*k}B^k = q^{k(1-k)/2}\eta^{2k}. \tag{71}$$

References

[1] Podleś P 1987 *Lett. Math. Phys.* **14** 193
[2] Manin Yu I 1988 Quantum groups and noncommutative geometry *Publ. CRM* vol 1561 University of Monreal
[3] Woronowicz S L 1987 *Commun. Math. Phys.* **111** 613
[4] Woronowicz S L 1991 *Commun. Math. Phys.* **136** 399
Woronowicz S L 1992 *Commun. Math. Phys.* **144** 417
Woronowicz S L 1992 *Commun. Math. Phys.* **149** 637
Woronowicz S L 1991 *Lett. Math. Phys.* **23** 251
[5] Vaksman L L and Korogodski L I 1989 *Dokl. Akad. Nauk SSSR* **304** 1036
[6] Bonechi F, Ciccoli N, Giaretti R, Sorace E and Tarlini M 1996 *Commun. Math. Phys.* **175** 161
[7] Koelink H T 1994 *Duke Math. J.* **76** 483
[8] Koornwinder T H 1989 *Nederl. Akad. Wetensch. Proc.* A **76** 97
Koornwinder T H 1991 *SIAM J. Math. Anal.* **22** 295
[9] Vilenkin N Ya and Klimyk A U 1991 *Representations of Lie Groups and Special Functions* vol 3 (Dordrecht: Kluwer)
[10] Ahmedov H and Duru I H 1998 *J. Phys. A: Math. Gen.* **31** 5741
[11] Ahmedov H and Duru I H 1999 *J. Phys. A: Math. Gen.* **32** 6255
[12] Ahmedov H and Duru I H 2000 *J. Phys. A: Math. Gen.* **33** 4277
[13] Ahmedov H and Duru I H 2001 *J. Phys. A: Math. Gen.* **34** 227
(Ahmedov H and Duru I H 2000 *Preprint* math. QA/0003078)
[14] Askey R and Suslov S K 1993 *J. Phys. A: Math. Gen.* **26** L693
[15] Askey R and Suslov S K 1993 *Lett. Math. Phys.* **29** 123
[16] Floreanini R and Vinet L 1993 *Phys. Lett.* A **180** 393
[17] Kalnins E G, Manocha H L and Miller W 1992 *J. Math. Phys.* **33** 2365
[18] Kalnins E G and Miller W 1994 *J. Math. Phys.* **35** 1951
[19] Kalnins E G, Miller W and Mukherjee S 1993 *J. Math. Phys.* **34** 5333
[20] Fronsdal C and Galingo A 1993 *Lett. Math. Phys.* **27** 59
[21] Koelink H T 1996 *J. Comput. Appl. Math.* **68** 209
[22] Moak D S 1981 *J. Math. Anal. Appl.* **81** 20
[23] Koornwinder T H and Swarttouw R F 1992 *Trans. Am. Math. Soc* **333** 445
[24] Gradshtein I S and Ryzhik I M 1980 *Tables of Integrals, Series and Products* (New York: Academic)